ECS 452: Digital Communication Systems **2013/1**

HW 5 — Due: Not Due

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Instructions

(a) Have fun!

Problem 1. Consider a standard rectangular 8-ary constellation shown in Figure 1. As usual, it is derived from the waveform models whose noise process is additive white Gaussian noise (AWGN) with PSD $\frac{N_0}{2} = 3$. The constellation is centered at the origin (so that the average E_s is minimized.) The vertical distances and horizontal distances between any adjacent points are all $d = 1$. Minimum-distance detector is used.

(a) Find the (1,2) element in the Q matrix. (This is the probability that the detector output is $\hat{W} = 2$ given that the actual intended message is $W = 1$.)

(b) Find the (3,5) element in the Q matrix.

(c) Find the value of $\frac{E_b}{N_0}$ for this constellation under the above description of noise. Assume equiprobable messages.

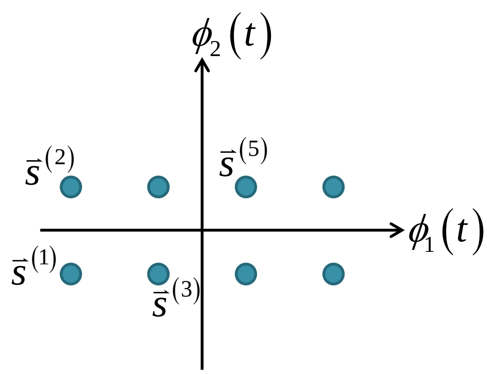

Figure 5.1: Constellations for Problem 1.

Problem 2. Often, we have to work with constellation that is difficult to derive the Q matrix (because the integrations involved are difficult. It's best to try to reduce the number of calculations that are needed.

In class, we have seen that, for QPSK, even though there are $4^2 = 16$ possible elements in the matrix Q, we only have to identify three elements in there. Here, consider the constellations in Figure 5.2.i and Figure 5.2.ii. Let's suppose that you have already calculated some elements of their Q matrices to be

$$Q = \left[\begin{array}{c|c|c} & 0.30 & \\ \hline & & \\ \hline & & \end{array}\right] \text{ and } Q = \left[\begin{array}{c|c|c|c} 0.41 & 0.29 & & \\ \hline & & & \\ \hline & & & \\ \hline & & 0.33 & \end{array}\right],$$

respectively.

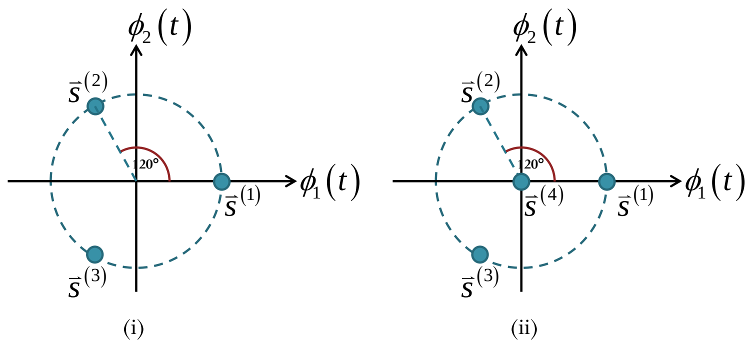

Figure 5.2: Constellations for Problem 2.

(a) Find the values of the rest of the elements. Assume minimum-distance (maximum-likelihood) decoder and AWGN channel.

(b) Find the (overall average) probability of (detection) error for each constellation. Assume that the points are equally likely.

Problem 3. Consider the vector channel derived from waveform channels under AWGN with PSD $\frac{N_0}{2}$. We consider two digital modulation: BPSK and QPSK. The detector at the receiver uses minimum-distance detection.

(a) Derive the formula and then plot the capacity of BPSK as a function of $\frac{E_b}{N_0}$.

(b) Derive the formula and then plot the capacity of QPSK as a function of $\frac{E_b}{N_0}$.

Problem 4. A linear block code has a generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

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(a) List all codewords for this code.

(b) Determine a suitable parity check matrix H.

(c) Check that $GH^T = 0$

(d) Find the minimum distance of this code.

Problem 5. A Hamming code has the parity check matrix given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

(a) What is the number of parity bits used in this code?

(b) Find the corresponding generator matrix.

(c) The following information bits are to be encoded using the Hamming code above:

001110111010

(i) How should the bits be split into blocks? In particular, what is the length of each block and how many blocks are used?

(ii) Find the corresponding codewords

(d) Some more information bits were generated. They were encoded using the Hamming code above. Suppose the received bits are

101101110000010100001

(i) How should the received bits be split into blocks (received vectors)? In particular, what is the length of each block and how many blocks are there?

(ii) Locate and correct all errors.